

# Ramsey numbers of 3-uniform loose paths and loose cycles

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## Abstract

Haxell et. al. [The Ramsey number for hypergraph cycles I, J. Combin. Theory, Ser. A, 113 (2006), 67-83] proved that the 2-color Ramsey number of 3-uniform loose cycles on  $2n$  vertices is asymptotically  $\frac{5n}{2}$ . Their proof is based on the method of Regularity Lemma. Here, without using this method, we generalize their result by determining the exact values of 2-color Ramsey numbers involving loose paths and cycles in 3-uniform hypergraphs. More precisely, we prove that for every  $n \geq m \geq 3$ ,  $R(\mathcal{P}_n^3, \mathcal{P}_m^3) = R(\mathcal{P}_n^3, \mathcal{C}_m^3) = R(\mathcal{C}_n^3, \mathcal{C}_m^3) + 1 = 2n + \lfloor \frac{m+1}{2} \rfloor$  and for  $n > m \geq 3$ ,  $R(\mathcal{P}_m^3, \mathcal{C}_n^3) = 2n + \lfloor \frac{m-1}{2} \rfloor$ . These give a positive answer to a question of Gyárfás and Raeisi [The Ramsey number of loose triangles and quadrangles in hypergraphs, Electron. J. Combin. 19 (2012), #R30].

**Keywords:** Ramsey number, Uniform hypergraph, Loose path, Loose cycle.

AMS subject classification: 05C65, 05C55, 05D10.

## 1 Introduction

A  $k$ -uniform *loose cycle*  $\mathcal{C}_n^k$  (shortly, a *cycle of length  $n$* ) is a hypergraph with vertex set  $\{v_1, v_2, \dots, v_{n(k-1)}\}$  and with the set of  $n$  edges  $e_i = \{v_1, v_2, \dots, v_k\} + i(k-1)$ ,  $i = 0, 1, \dots, n-1$ , where we use mod  $n(k-1)$  arithmetic and adding a number  $t$  to a set  $H = \{v_1, v_2, \dots, v_k\}$  means a shift, i.e. the set obtained by adding  $t$  to subscripts of each element of  $H$ . Similarly, a  $k$ -uniform *loose path*  $\mathcal{P}_n^k$  (simply, a *path of length  $n$* ) is a hypergraph with vertex set  $\{v_1, v_2, \dots, v_{n(k-1)+1}\}$  and with the set of  $n$  edges  $e_i = \{v_1, v_2, \dots, v_k\} + i(k-1)$ ,  $i = 0, 1, \dots, n-1$ . For  $k = 2$  we get the usual definitions of a cycle  $C_n$  and a path  $P_n$  with  $n$  edges. For an edge  $e$  of a given loose path (also a given loose cycle)  $\mathcal{K}$ , the first vertex and the last vertex are denoted by  $v_{\mathcal{K},e}$  and  $\hat{v}_{\mathcal{K},e}$ , respectively. For two  $k$ -uniform hypergraphs  $\mathcal{H}$  and  $\mathcal{G}$ , the *Ramsey number*  $R(\mathcal{H}, \mathcal{G})$  is the smallest number  $N$  such that each red-blue coloring of the edges of the complete  $k$ -uniform hypergraph  $\mathcal{K}_N^k$  on  $N$  vertices contains either a red copy of  $\mathcal{H}$  or a blue copy of  $\mathcal{G}$ .

One of the central problems in combinatorics and graph theory is determining or estimating the Ramsey numbers. In this area, a classical topic is the study of the

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Ramsey numbers of sparse graphs, i.e., graphs with certain upper bound constraints on the degrees of the vertices. The study of these Ramsey numbers was initiated by Burr and Erdős [2]. They conjectured that for integer  $\Delta$  there is a constant  $c = c(\Delta)$  so that for every graph  $G$  on  $n$  vertices and maximum degree  $\Delta$ ,  $R(G, G) \leq cn$ . This conjecture was established by Chvátal et. al. in 1983 [3]. In this area the exact values of  $R(P_n, P_m)$ ,  $R(P_n, C_m)$  and  $R(C_n, C_m)$  are determined (see [1, 6, 7, 8, 16]). For a recent survey including some results on the Ramsey numbers of sparse graphs see [15].

It was natural to extend the results on the Ramsey number of sparse graphs for hypergraphs. In this direction, Kostochka and Rödl [12] had shown that for every  $\epsilon > 0$  and every positive integers  $\Delta$  and  $r$ , there exists  $c = c(\epsilon, \Delta, r)$  such that  $R(\mathcal{H}, \mathcal{H})$  for any  $r$ -uniform hypergraph  $\mathcal{H}$  with maximum degree at most  $\Delta$  is at most  $c|V(\mathcal{H})|^{1+\epsilon}$ ; that is, the Ramsey numbers of  $k$ -uniform hypergraphs of bounded maximum degree are almost linear in their orders. In a sequel, Nagle et. al. [14] and Cooley et. al. [5] independently showed that the Ramsey numbers of 3-uniform hypergraphs of bounded maximum degree are linear in their order. This result was extended by several authors to  $k$ -uniform hypergraphs. For an excellent article giving the latest results in this direction, we refer the reader to [4]. Related with this topic, several interesting results were obtained on 2-color Ramsey numbers of loose cycles and paths in uniform hypergraphs. Haxell et. al. [11] showed the following result on the Ramsey number of 3-uniform loose cycles, using Regularity Lemma.

**Theorem 1.1** *For all  $\eta > 0$  there exists  $n_0 = n_0(\eta)$  such that for every  $n > n_0$ , every 2-colouring of  $\mathcal{K}_{5(1+\eta)n/2}^3$  contains a monochromatic copy of  $C_n^3$ .*

Subsequently, Gyárfás, Sárközy and Szemerédi [9] extended this result to  $k$ -uniform loose cycles and proved that for  $k \geq 3$  and all  $\eta > 0$  there exists  $n_0 = n_0(\eta)$  such that every 2-coloring of  $\mathcal{K}_N^k$  with  $N = (1 + \eta)\frac{1}{2}(2k - 1)n$  contains a monochromatic copy of  $C_n^k$ , i.e.  $R(C_n^k, C_n^k)$  is asymptotically equal to  $\frac{1}{2}(2k - 1)n$ . However, those proofs are all based on application of the hypergraph regularity method. Recently, Gyárfás and Raeisi determined the exact values of 2-color Ramsey numbers of two  $k$ -uniform loose triangles and two  $k$ -uniform loose quadrangles [10]. They also posed the following question.

**Question 1.2** *For every  $n \geq m \geq 3$ , is it true that  $R(\mathcal{P}_n^3, \mathcal{P}_m^3) = R(\mathcal{P}_n^3, C_m^3) = R(C_n^3, C_m^3) + 1 = 2n + \lfloor \frac{m+1}{2} \rfloor$ ? In particular,*

$$R(\mathcal{P}_n^3, \mathcal{P}_n^3) = R(C_n^3, C_n^3) + 1 = \left\lceil \frac{5n}{2} \right\rceil?$$

Related to question 1.2, it is proved [13] that for every  $n \geq \lfloor \frac{5m}{4} \rfloor$ ,  $R(\mathcal{P}_n^3, \mathcal{P}_m^3) = 2n + \lfloor \frac{m+1}{2} \rfloor$ . In this article, with a much short proof and without using the Regularity Lemma, we give a positive answer to this question. Moreover, we show that  $R(\mathcal{P}_m^3, C_n^3) = 2n + \lfloor \frac{m-1}{2} \rfloor$  for  $n > m \geq 3$ . Indeed, our results yield Theorem 1.1.

The rest of this paper is organized as follows. In the next section, we state the principal results required to prove the main results. In section 3, we determine the

exact value of the Ramsey number of loose cycles in 3-uniform hypergraphs which is a generalization of Theorem 1.1. In section 4, we provide the exact value of the Ramsey number of loose paths in 3-uniform hypergraphs and finally in the last section, the Ramsey number of a loose path and a loose cycle in 3-uniform hypergraphs is determined.

**Note:** It is shown in [10] that  $(k-1)n + \lfloor \frac{m+1}{2} \rfloor$ ,  $n \geq m \geq 2$  and  $k \geq 3$ , is a lower bound for  $R(\mathcal{P}_n^k, \mathcal{P}_m^k)$ ,  $R(\mathcal{P}_n^k, \mathcal{C}_m^k)$  and  $R(\mathcal{C}_n^k, \mathcal{C}_m^k) + 1$ . Here we mention that for  $n > m$  and  $k \geq 3$ ,  $R(\mathcal{P}_m^k, \mathcal{C}_n^k) \geq (k-1)n + \lfloor \frac{m-1}{2} \rfloor$ . To see that consider a complete hypergraph whose vertex set is partitioned into two parts  $A$  and  $B$ , where  $|A| = (k-1)n - 1$  and  $|B| = \lfloor \frac{m-1}{2} \rfloor$ . Color all edges that contain a vertex of  $B$  red, and the rest blue. In this coloring, the longest red path has length at most  $m-1$  and there is also no blue copy of  $\mathcal{C}_n^k$ , since such a copy has  $(k-1)n$  vertices. Our main aim in this article is to prove that these lower bounds are tight. Therefore, in the rest of this paper, we shall prove just upper bounds for the claimed Ramsey numbers. Throughout the paper, we denote by  $\mathcal{H}_{red}$  and  $\mathcal{H}_{blue}$  the induced 3-uniform hypergraph on edges of color red and blue, respectively. Also we denote by  $|\mathcal{H}|$  and  $\|\mathcal{H}\|$  the number of vertices and edges of  $\mathcal{H}$ , respectively.

## 2 Preliminaries

In this section, we present some useful results. We also rewrite some results from [10] in below.

**Lemma 2.1** [10] *Let  $n \geq m \geq 3$  and  $\mathcal{K}_{(k-1)n + \lfloor \frac{m+1}{2} \rfloor}^k$  be 2-edge colored red and blue. If  $\mathcal{C}_n^k \subseteq \mathcal{H}_{red}$ , then either  $\mathcal{P}_n^k \subseteq \mathcal{H}_{red}$  or  $\mathcal{P}_m^k \subseteq \mathcal{H}_{blue}$ . Also, if  $\mathcal{C}_n^k \subseteq \mathcal{H}_{red}$  then either  $\mathcal{P}_n^k \subseteq \mathcal{H}_{red}$  or  $\mathcal{C}_m^k \subseteq \mathcal{H}_{blue}$ .*

**Theorem 2.2** [10] *For every  $k \geq 3$ ,*

$$I) R(\mathcal{P}_3^k, \mathcal{P}_3^k) = R(\mathcal{C}_3^k, \mathcal{P}_3^k) = R(\mathcal{C}_3^k, \mathcal{C}_3^k) + 1 = 3k - 1.$$

$$II) R(\mathcal{P}_4^k, \mathcal{P}_4^k) = R(\mathcal{C}_4^k, \mathcal{P}_4^k) = R(\mathcal{C}_4^k, \mathcal{C}_4^k) + 1 = 4k - 2.$$

Let  $\mathcal{P}$  be a loose path and  $W$  be a set of vertices with  $W \cap V(\mathcal{P}) = \emptyset$ . By a  $\varpi_{\{v_i, v_j, v_k\}}$ -configuration, we mean a copy of  $\mathcal{P}_2^3$  with edges  $\{x, v_i, v_j\}$  and  $\{v_j, v_k, y\}$  so that  $v_l$ 's,  $l \in \{i, j, k\}$ , belong to two consecutive edges of  $\mathcal{P}$  and  $\{x, y\} \subseteq W$ . The vertices  $x$  and  $y$  are called the end vertices of this configuration. A  $\varpi_S$ -configuration,  $S \subseteq V(e_i) \cup V(e_{i+1})$ , is good if the last vertex of  $e_{i+1}$  is not in  $S$  and it is bad, otherwise. Let  $\mathcal{H} = \mathcal{K}_l^k$  be 2-edge colored red and blue. We say that a red path  $\mathcal{P} = e_1 e_2 \dots e_n$  of length  $n$  is *maximal with respect to  $W$*  (for abbreviation w.r.t.  $W$ ), for  $W \subseteq V(\mathcal{H}) \setminus V(\mathcal{P})$ , if there are no vertices  $x$  and  $y$  in  $W$  so that for some  $1 \leq i \leq n$  either there is a red path  $\mathcal{P}' = e_1 e_2 \dots e_{i-1} e e' e_{i+1} \dots e_n$  with  $v_{\mathcal{P}', e} = v_{\mathcal{P}, e_i}$  for  $i = 1$  and  $\hat{v}_{\mathcal{P}', e'} = \hat{v}_{\mathcal{P}, e_i}$  for  $i = n$ , or a red path  $\mathcal{P}' = e_1 e_2 \dots e_{i-1} e e' e'' e_{i+2} \dots e_n$  with  $v_{\mathcal{P}', e} = v_{\mathcal{P}, e_i}$  for  $i = 1$  and  $\hat{v}_{\mathcal{P}', e''} = \hat{v}_{\mathcal{P}, e_{i+1}}$  for  $i = n-1$ , in  $\mathcal{H}$  so that  $V(\mathcal{P}') = V(\mathcal{P}) \cup \{x, y\}$ . We use these definitions to deduce the following Lemma.

**Lemma 2.3** Assume that  $\mathcal{H} = \mathcal{K}_n^3$  is 2-edge colored red and blue. Let  $\mathcal{P} \subseteq \mathcal{H}_{\text{red}}$  be maximal w.r.t.  $W \subseteq V(\mathcal{H}) \setminus V(\mathcal{P})$  with  $|W| \geq 3$ . Let  $e_i = \{v_{2i-1}, v_{2i}, v_{2i+1}\}$  and  $e_{i+1} = \{v_{2i+1}, v_{2i+2}, v_{2i+3}\}$  be consecutive edges of  $\mathcal{P}$ . Either there is a good  $\varpi_S$ -configuration  $C$  in  $\mathcal{H}_{\text{blue}}$  with the set of end vertices in  $W$  and  $S \subseteq V(e_i) \cup V(e_{i+1})$  or there is a bad  $\varpi_{S_1}$ -configuration  $C_1$ ,  $S_1 \subseteq V(e_i) \cup V(e_{i+1}) \setminus \{v_{2i+2}\}$ . If there is no such a good configuration  $C$  in  $\mathcal{H}_{\text{blue}}$  and  $e_{i+2} = \{v_{2i+3}, v_{2i+4}, v_{2i+5}\}$  is an edge of  $\mathcal{P}$ , then also there is a good  $\varpi_{S_2}$ -configuration  $C_2$ ,  $S_2 \subseteq V(e_{i+1}) \cup V(e_{i+2})$ , in  $\mathcal{H}_{\text{blue}}$  with end vertices in  $W$  and  $S_1 \cap S_2 = \emptyset$ . Moreover, each vertex of  $W$  except at most one vertex can be considered as an end vertex of  $C$  if there exists such a configuration and otherwise, each vertex of  $W$  can be considered as an end vertex of  $C_1$  and  $C_2$ .

**Proof.** Assume  $W = \{x_1, \dots, x_t\} \subseteq V(\mathcal{K}_n^3) \setminus V(\mathcal{P})$ . If for some  $x \in W$ ,  $\{v_{2i-1}, v_{2i}, x\}$  (resp.  $\{v_{2i+2}, v_{2i+3}, x\}$ ) is red, then the maximality of  $\mathcal{P}$  w.r.t.  $W$  implies that for arbitrary vertices  $x' \neq x'' \in W \setminus \{x\}$  the edges  $\{x', v_{2i+1}, v_{2i}\}$  and  $\{v_{2i}, v_{2i+2}, x''\}$  (resp.  $\{x', v_{2i+1}, v_{2i+2}\}$  and  $\{v_{2i+2}, v_{2i}, x''\}$ ) are blue and there is a good  $\varpi_S$ -configuration  $C = \{x', v_{2i+1}, v_{2i}\}\{v_{2i}, v_{2i+2}, x''\}$  (resp.  $C = \{x', v_{2i+1}, v_{2i+2}\}\{v_{2i+2}, v_{2i}, x''\}$ ) with  $S = \{v_{2i}, v_{2i+1}, v_{2i+2}\} \subseteq V(e_i) \cup V(e_{i+1})$ . So we may assume that for every  $x \in W$  both edges  $\{v_{2i-1}, v_{2i}, x\}$  and  $\{v_{2i+2}, v_{2i+3}, x\}$  are blue. If there is a vertex  $y \in W$  such that at least one of the edges  $f_1 = \{v_{2i-1}, v_{2i+1}, y\}$ ,  $f_2 = \{v_{2i}, v_{2i+1}, y\}$ ,  $f_3 = \{v_{2i-1}, v_{2i+2}, y\}$  or  $f_4 = \{v_{2i}, v_{2i+2}, y\}$ , say  $f$ , is blue, then there is a good  $\varpi_S$ -configuration  $C = \{v_{2i-1}, v_{2i}, x\}f$ ,  $x \neq y$ , with  $S = \{v_{2i-1}, v_{2i}\} \cup f \setminus \{y\} \subseteq V(e_i) \cup V(e_{i+1})$ . So if there is no such a good configuration  $C$ , then we may assume that for every  $y \in W$  the edges  $f_1, f_2, f_3$  and  $f_4$  are red. Therefore, maximality of  $\mathcal{P}$  w.r.t.  $W$  implies that for every  $y' \in W$  the edge  $\{v_{2i}, v_{2i+3}, y'\}$  is blue (otherwise, replacing  $e_i e_{i+1}$  by  $\{v_{2i-1}, v_{2i+2}, y'\}\{y, v_{2i+1}, v_{2i}\}\{v_{2i}, v_{2i+3}, y'\}$ ,  $y \neq y'$ , in  $\mathcal{P}$  yields a red path  $\mathcal{P}'$  greater than  $\mathcal{P}$ , a contradiction). So, for every  $a \neq b \in W$ ,  $C_1 = \{v_{2i-1}, a, v_{2i}\}\{v_{2i}, v_{2i+3}, b\}$  is a bad  $\varpi_{S_1}$ -configuration with desired properties so that  $S_1 = \{v_{2i-1}, v_{2i}, v_{2i+3}\} \subseteq V(e_i) \cup V(e_{i+1}) \setminus \{v_{2i+2}\}$ . Clearly if there is  $e_{i+2} = \{v_{2i+3}, v_{2i+4}, v_{2i+5}\}$  as an edge of  $\mathcal{P}$ , then for every  $x \in W$ ,  $\{v_{2i+2}, v_{2i+4}, x\}$  (also  $\{v_{2i+1}, v_{2i+4}, x\}$ ) is blue, otherwise replacing  $e_i e_{i+1}$  by  $\{v_{2i-1}, v_{2i+1}, y\}\{y, v_{2i}, v_{2i+2}\}\{v_{2i+2}, v_{2i+4}, x\}$  (replacing  $e_i e_{i+1}$  by  $\{v_{2i-1}, v_{2i+2}, y\}\{y, v_{2i}, v_{2i+1}\}\{v_{2i+1}, v_{2i+4}, x\}$ ) for some  $y \neq x$  in  $\mathcal{P}$  yields a red path  $\mathcal{P}'$  greater than  $\mathcal{P}$ , a contradiction to the maximality of  $\mathcal{P}$  w.r.t.  $W$ . So for every  $a \neq b \in W$ ,  $C_2 = \{a, v_{2i+2}, v_{2i+4}\}\{v_{2i+4}, v_{2i+1}, b\}$  is a good  $\varpi_{S_2}$ -configuration with desired properties where  $S_2 = \{v_{2i+1}, v_{2i+2}, v_{2i+4}\} \subseteq V(e_{i+1}) \cup V(e_{i+2})$ . ■

The following is an immediate corollary of the Lemma 2.3.

**Corollary 2.4** Assume that  $\mathcal{H} = \mathcal{K}_n^3$  is 2-edge colored red and blue. Let  $\mathcal{P} \subseteq \mathcal{H}_{\text{red}}$  be maximal w.r.t.  $W \subseteq V(\mathcal{H}) \setminus V(\mathcal{P})$  with  $|W| \geq 3$ . Let  $e_i = \{v_{2i-1}, v_{2i}, v_{2i+1}\}$ ,  $e_{i+1} = \{v_{2i+1}, v_{2i+2}, v_{2i+3}\}$  and  $e_{i+2} = \{v_{2i+3}, v_{2i+4}, v_{2i+5}\}$  be consecutive edges of  $\mathcal{P}$ . Either for every distinct vertices  $x$  and  $y$  of  $W$ , except at most one vertex there is a blue path  $Q = \{\bar{v}_1, x, \bar{v}_2\}\{\bar{v}_2, y, \bar{v}_3\}$  of length 2 with  $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\} \subseteq V(e_i) \cup V(e_{i+1}) \setminus \{v_{2i+3}\}$  or for every distinct vertices  $x, y$  and  $z$  of  $W$  there is a blue path  $Q' = \{v'_1, x, v'_2\}\{v'_2, v'_3, y\}\{y, v'_4, v'_5\}\{v'_5, v'_6, z\}$  of length 4 with  $\{v'_1, v'_2, v'_3, v'_4, v'_5, v'_6\} \subseteq V(e_i) \cup V(e_{i+1}) \cup V(e_{i+2}) \setminus \{v_{2i+5}\}$ .

**Corollary 2.5** *Let  $\mathcal{H} = \mathcal{K}_l^3$  be two edge colored red and blue and  $\mathcal{P} = e_1 e_2 \dots e_n$  be maximal red path w.r.t.  $W$ ,  $W \subseteq V(\mathcal{H}) \setminus V(\mathcal{P})$ . Then for some  $r \geq 0$  and  $W' \subseteq W$  there is a blue path  $Q$  between  $W'$  and  $\bar{\mathcal{P}} = e_1 e_2 \dots e_{n-r}$  with no  $\hat{v}_{\mathcal{P}, e_{n-r}}$  as a vertex and  $\|Q\| = 2(|W'| - 1) \geq n - r$  where either  $x = |W \setminus W'| = 0, 1$  or  $x \geq 2$  and  $0 \leq r \leq 2$ .*

**Proof.** Let  $\mathcal{P} = e_1 e_2 \dots e_n$  be a maximal red path w.r.t.  $W = V(\mathcal{H}) \setminus V(\mathcal{P})$  where  $e_i = \{v_1, v_2, v_3\} + 2(i-1)$ ,  $i = 1, \dots, n$ . Since  $\mathcal{P}_1 = \mathcal{P}$  is maximal w.r.t.  $W_1 = W$ , using Corollary 2.4 there is a blue path  $Q_1$  with end vertices  $x_1$  and  $y_1$  in  $W_1$  between  $E'_1$  and  $W_1$  where  $E_1 = \{e_i : i_1 = 1 \leq i \leq 2\}$ ,  $\bar{E}_1 = E_1 \cup \{e_{i_1+2}\}$ ,  $E'_1 \in \{E_1, \bar{E}_1\}$ ,  $\|Q_1\| = 2|E'_1| - 2$  and  $Q_1$  does not contain the last vertex of  $E'_1$ . Set  $i_2 = \min\{j : j \in \{i_1 + 2, i_1 + 3\}, e_j \notin E'_1\}$ ,  $E_2 = \{e_i : i_2 \leq i \leq i_2 + 1\}$ ,  $\bar{E}_2 = E_2 \cup \{e_{i_2+2}\}$ ,  $\mathcal{P}_2 = \mathcal{P}_1 \setminus E'_1$  and  $W_2 = (W \setminus V(Q_1)) \cup \{x_1, y_1\}$ . Since  $\mathcal{P}_2$  is maximal w.r.t.  $W_2$ , using Corollary 2.4 there is a blue path  $Q_2$  between  $E'_2$ ,  $E'_2 \in \{E_2, \bar{E}_2\}$ , and  $W_2$  such that  $\|Q_2\| = 2|E'_2| - 2$  and  $Q_2$  does not contain the last vertex of  $E'_2$  and  $Q_1 \cup Q_2$  is a blue path with end vertices  $x_2, y_2$  in  $W_2$ . Now set  $i_3 = \min\{j : j \in \{i_2 + 2, i_2 + 3\}, e_j \notin E'_2\}$ ,  $E_3 = \{e_i : i_3 \leq i \leq i_3 + 1\}$ ,  $\bar{E}_3 = E_3 \cup \{e_{i_3+2}\}$ ,  $\mathcal{P}_3 = \mathcal{P}_2 \setminus E'_2$  and  $W_3 = (W \setminus V(Q_1 \cup Q_2)) \cup \{x_2, y_2\}$  and continue this process. Assume that this process is terminated in  $t$ -th step. Clearly either  $|W \setminus W_t| = 0, 1$  or  $|W \setminus W_t| \geq 2$  and  $0 \leq \|\mathcal{P}_t\| \leq 2$ . So  $Q = Q_1 \cup Q_2 \cup \dots \cup Q_{t-1}$  is a blue path between  $\bar{\mathcal{P}} = e_1 e_2 \dots e_{n-r}$  and  $W' = (W \setminus W_t) \cup \{x_{t-1}, y_{t-1}\}$  where  $\mathcal{P}_t = e_{n-r+1} e_{n-r+2} \dots e_n$ . ■

### 3 Cycle-cycle Ramsey number in 3-uniform hypergraphs

In this section, we provide the exact value of  $R(\mathcal{C}_n^3, \mathcal{C}_m^3)$ , when  $n \geq m \geq 3$ . Before that we need two following lemmas.

**Lemma 3.1** *Let  $n \geq m \geq 3$ ,  $(n, m) \neq (3, 3), (4, 3), (4, 4)$  and  $\mathcal{K}_{2n + \lfloor \frac{m-1}{2} \rfloor}^3$  be 2-edge colored red and blue. Assume there is no copy of  $\mathcal{C}_n^3$  in  $\mathcal{H}_{red}$  and  $\mathcal{C} = \mathcal{C}_{n-1}^3$  is a loose cycle in  $\mathcal{H}_{red}$ . Then there is a copy of  $\mathcal{C}_m^3$  in  $\mathcal{H}_{blue}$ . Moreover, for  $n > m$ , there is also a copy of  $\mathcal{P}_m^3$  in  $\mathcal{H}_{blue}$ .*

**Proof.** Let  $t = 2n + \lfloor \frac{m-1}{2} \rfloor$  and  $\mathcal{C} = e_1 e_2 \dots e_{n-1}$  be a copy of  $\mathcal{C}_{n-1}^3$  in  $\mathcal{H}_{red}$  with edges  $e_i = \{v_1, v_2, v_3\} + 2(i-1) \pmod{2(n-1)}$ ,  $i = 1, \dots, n-1$ . Let  $W = V(\mathcal{H}) \setminus V(\mathcal{C})$  where  $\mathcal{H} = \mathcal{K}_t^3$ .

**Case 1.** There are an edge  $e_i = \{v_{2i-1}, v_{2i}, v_{2i+1}\}$ ,  $1 \leq i \leq n-1$ , and a vertex  $z \in W$  such that  $\{v_{2i}, v_{2i+1}, z\}$  is red.

Let  $\mathcal{P} = e_{i+1} e_{i+2} \dots e_{n-1} e_1 e_2 \dots e_{i-2} e_{i-1}$  and  $W_0 = W \setminus \{z\}$ . Since  $\mathcal{P}$  is a maximal path w.r.t.  $W_0$ , using Corollary 2.5, there is a blue path  $Q$  of length  $l'$  between  $\bar{\mathcal{P}}$ , the path obtained from  $\mathcal{P}$  by deleting the last  $r$  edges, and  $W'$  for some  $r \geq 0$  and  $W' \subseteq W_0$  with mentioned properties in Corollary 2.5. Let  $x', y'$  be the end vertices of  $Q$  in  $W'$ ,  $T = W_0 \setminus V(Q)$  and  $x = |T|$ . We have following subcases.

*Subcase 1.  $x = 0$ .*

Clearly  $l' = 2\lfloor \frac{m-1}{2} \rfloor$ . First let  $m$  be even. Hence  $l' = m-2$  and so  $Q\{y', v_{2i}, v_{2i-1}\} \{v_{2i-1}, z, x'\}$  is a blue  $\mathcal{C}_m^3$ . Moreover, if  $n > m$ , then  $r \geq 1$  and so at least one

of  $\{v_{2i-3}, v_{2i-2}, x'\}Q\{y', v_{2i-1}, v_{2i}\}$  or  $Q\{y', v_{2i-1}, v_{2i}\}\{v_{2i}, v_{2i-2}, z\}$  is a blue copy of  $\mathcal{P}_m^3$ . Now let  $m$  be odd. So  $l' = m - 1$ . In this case we truncate  $Q$  to a path  $Q'$  of length  $m - 3$  so that  $v_{2i-2} \notin Q'$  by removing the last two edges. Now we may assume the vertices  $x'$  and  $y'' \neq y'$  of  $W_0$  are the end vertices of  $Q'$ . So  $Q'\{y'', v_{2i-2}, v_{2i}\}\{v_{2i}, y', v_{2i-1}\}\{v_{2i-1}, z, x'\}$  is a copy of  $\mathcal{C}_m^3$  in  $\mathcal{H}_{blue}$ . Also  $Q\{y', v_{2i-1}, v_{2i}\}$  is a blue copy of  $\mathcal{P}_m^3$ .

*Subcase 2.  $x = 1$ .*

Let  $T = \{u\}$ . Clearly  $l' = 2\lfloor \frac{m-1}{2} \rfloor - 2$ . Let  $m$  be odd. Then  $l' = m - 3$  and  $r \geq 1$ . So  $Q\{y', v_{2i-2}, v_{2i}\}\{v_{2i}, u, v_{2i-1}\}\{v_{2i-1}, z, x'\}$  is a blue copy of  $\mathcal{C}_m^3$ . If  $n > m$ , then  $r \geq 2$  and since  $\hat{\mathcal{P}} = e_{i-2}e_{i-1}e_i$  is a maximal path w.r.t.  $\hat{W} = \{x', y', u, z\}$ , using Corollary 2.4 there is either a blue path  $Q'$  of length 2 between  $(V(e_{i-2}) \cup V(e_{i-1})) \setminus \{v_{2i-1}\}$  and  $\hat{W}$  or a blue path  $Q'$  of length 4 between  $V(\hat{\mathcal{P}}) \setminus \{v_{2i+1}\}$  and  $\hat{W}$  so that  $Q \cup Q'$  is a blue path, say  $Q''$ . Now let  $l''$  be the length of  $Q'$ . If  $l'' = 4$ ,  $Q''$  is a blue  $\mathcal{P}_{m+1}^3$  and so there is a  $\mathcal{P}_m^3$  in  $\mathcal{H}_{blue}$ . If  $l'' = 2$ , the length of  $Q''$  is  $m - 1$ . Without loss of generality let  $x', y''$  be the end vertices of  $Q''$ . Then  $\{v_{2i-1}, v_{2i}, x'\}Q''$  is a blue copy of  $\mathcal{P}_m^3$ . Now suppose  $m$  is even, so  $l' = m - 4$  and  $r \geq 2$ . If  $\{v_{2i-5}, v_{2i-4}, x'\}$  is red, then  $Q\{y', v_{2i-3}, v_{2i-4}\}\{v_{2i-4}, v_{2i-2}, u\}\{u, v_{2i}, v_{2i-1}\}\{v_{2i-1}, z, x'\}$  is a blue  $\mathcal{C}_m^3$ . If  $\{v_{2i-4}, v_{2i-3}, u\}$  is red, then  $Q\{y', v_{2i-1}, v_{2i}\}\{v_{2i}, v_{2i-2}, u\}\{u, z, v_{2i-5}\}\{v_{2i-5}, v_{2i-4}, x'\}$  is a blue copy of  $\mathcal{C}_m^3$ . Otherwise,  $Q\{y', v_{2i-1}, v_{2i}\}\{v_{2i}, v_{2i-2}, u\}\{u, v_{2i-3}, v_{2i-4}\}\{v_{2i-4}, v_{2i-5}, x'\}$  is a copy of  $\mathcal{C}_m^3$  in  $\mathcal{H}_{blue}$ . For  $n > m$ , clearly  $r \geq 3$  and since  $\hat{\mathcal{P}} = e_{i-3}e_{i-2}e_{i-1}$  is a maximal path w.r.t.  $\hat{W} = \{x', y', u, z\}$ , using Corollary 2.4 there is a blue path  $Q'$  of length either  $l'' = 2$ , between  $(V(e_{i-3}) \cup V(e_{i-2})) \setminus \{v_{2i-3}\}$  and  $\hat{W}$  or  $l'' = 4$ , between  $V(\hat{\mathcal{P}}) \setminus \{v_{2i-1}\}$  and  $\hat{W}$  such that  $Q \cup Q'$  is a blue path, say  $Q''$ . If  $l'' = 4$ , then  $Q''$  is a blue  $\mathcal{P}_m^3$ . otherwise the length of  $Q''$  is  $m - 2$  and we may assume that  $x', y''$  be the end vertices of  $Q''$ . If  $y'' = z$ , then  $\{u, v_{2i-2}, v_{2i}\}\{v_{2i}, v_{2i-1}, x'\}Q''$  is a blue copy of  $\mathcal{P}_m^3$ . Otherwise, at least one of  $\{v_{2i-3}, v_{2i-2}, x'\}Q''\{y'', v_{2i-1}, v_{2i}\}$  or  $Q''\{y'', v_{2i-1}, v_{2i}\}\{v_{2i}, v_{2i-2}, z\}$  is a blue copy of  $\mathcal{P}_m^3$ .

*Subcase 3.  $x \geq 2$ .*

One can easily check that  $r \geq 3$ . This case does not occur by Corollary 2.5.

**Case 2.** There are  $e_i = \{v_{2i-1}, v_{2i}, v_{2i+1}\}$ ,  $1 \leq i \leq n - 1$ , and a vertex  $z \in W$  such that  $\{v_{2i-1}, v_{2i}, z\}$  is red.

In this case, consider the path  $\mathcal{P} = e_{i-1}e_{i-2} \dots e_2e_1e_{n-1}e_{n-2} \dots e_{i+2}e_{i+1}$  and repeat the proof of case 1. By an argument similar to the case 1 we can find a blue copy of  $\mathcal{C}_m^3$  and a blue copy of  $\mathcal{P}_m^3$  for  $n > m$ .

**Case 3.** For every  $e_i = \{v_{2i-1}, v_{2i}, v_{2i+1}\}$ ,  $1 \leq i \leq n - 1$ , and every vertex  $z \in W$  the edges  $\{v_{2i-1}, v_{2i}, z\}$  and  $\{v_{2i}, v_{2i+1}, z\}$  are blue.

In this case, clearly there are blue copies of  $\mathcal{C}_m^3$  and  $\mathcal{P}_m^3$  between  $V(\mathcal{C})$  and  $W$  and these observations complete the proof.  $\blacksquare$

**Lemma 3.2**  $R(\mathcal{C}_4^3, \mathcal{C}_3^3) = 9$

**Proof.** Let  $\mathcal{H} = \mathcal{K}_9^3$  be 2-edge colored red and blue. Suppose that there is no red copy of  $\mathcal{C}_4^3$  and no blue copy of  $\mathcal{C}_3^3$ . Using Theorem 2.2 we may assume that there is a blue copy of  $\mathcal{C}_4^3$ . Let  $\mathcal{C} = e_1e_2e_3e_4$  be a copy of  $\mathcal{C}_4^3$  in  $\mathcal{H}_{blue}$  with edges  $e_i = \{v_1, v_2, v_3\} + 2(i-1) \pmod{8}$ ,  $i = 1, \dots, 4$ . Let  $v \in V(\mathcal{H}) \setminus V(\mathcal{C})$ . Since there is no blue copy of  $\mathcal{C}_3^3$ ,  $\{v_1, v_6, v\}\{v, v_3, v_8\}\{v_8, v_4, v_2\}\{v_2, v_5, v_1\}$  is a red copy of  $\mathcal{C}_4^3$ , a contradiction. ■

The main result of this section is the following result on the Ramsey number of loose cycles in 3-uniform hypergraphs.

**Theorem 3.3** *For every  $n \geq m \geq 3$ ,*

$$R(\mathcal{C}_n^3, \mathcal{C}_m^3) = 2n + \left\lfloor \frac{m-1}{2} \right\rfloor.$$

**Proof.** We prove this theorem by induction on  $m+n$ . Using Theorem 2.2, the base case is trivial. Suppose indirectly that the edges of  $\mathcal{H} = \mathcal{K}_{2n+\lfloor \frac{m-1}{2} \rfloor}^3$  can be colored red and blue without a red copy of  $\mathcal{C}_n^3$  and a blue copy of  $\mathcal{C}_m^3$ . Consider the following cases.

**Case 1.**  $n = m$

Using Theorem 2.2 and Lemma 3.2 we may assume that  $n \geq 5$ . By the induction hypothesis,  $R(\mathcal{C}_{n-1}^3, \mathcal{C}_{n-1}^3) = 2(n-1) + \left\lfloor \frac{n-2}{2} \right\rfloor < 2n + \left\lfloor \frac{n-1}{2} \right\rfloor$ . So we may assume that there is a red copy of  $\mathcal{C}_{n-1}^3$  in  $\mathcal{H}$ . Using Lemma 3.1 we have a blue  $\mathcal{C}_n^3$ , a contradiction.

**Case 2.**  $n > m$ .

In this case,  $n-1 \geq m$  and since  $R(\mathcal{C}_{n-1}^3, \mathcal{C}_m^3) = 2(n-1) + \left\lfloor \frac{m-1}{2} \right\rfloor < 2n + \left\lfloor \frac{m-1}{2} \right\rfloor$ , we may assume that  $\mathcal{C}_{n-1}^3 \subseteq \mathcal{H}_{red}$ . Using Lemmas 3.1 we have a blue  $\mathcal{C}_m^3$ , a contradiction. These observations complete the proof. ■

## 4 Path-path Ramsey number in 3-uniform hypergraphs

In this section, we determine the exact value of  $R(\mathcal{P}_n^3, \mathcal{P}_m^3)$ , for  $n \geq m \geq 3$ . To see that we need the following Lemmas.

**Lemma 4.1** *Let  $n \geq m \geq 3$ ,  $(n, m) \neq (3, 3), (4, 3), (4, 4)$  and  $\mathcal{K}_{2n+\lfloor \frac{m+1}{2} \rfloor}^3$  be 2-edge colored red and blue. If  $\mathcal{P} = \mathcal{P}_{n-1}^3$  is a maximum red path, then there is a copy of  $\mathcal{P}_m^3$  in  $\mathcal{H}_{blue}$ .*

**Proof.** Let  $t = 2n + \lfloor \frac{m+1}{2} \rfloor$ ,  $\mathcal{P} = e_1e_2 \dots e_{n-1}$  be a red copy of  $\mathcal{P}_{n-1}^3$  with edges  $e_i = \{v_1, v_2, v_3\} + 2(i-1)$ ,  $i = 1, \dots, n-1$  and  $\bar{W} = V(\mathcal{K}_t^3) \setminus V(\mathcal{P})$ . By Lemma 2.1, we may assume that there is no copy of  $\mathcal{C}_n^3$  in  $\mathcal{H}_{red}$ . Let  $W_0 = \bar{W} \setminus \{u\}$  for  $u \in \bar{W}$ . Since  $\mathcal{P}' = \mathcal{P} \setminus \{e_1\}$  is a maximal path w.r.t.  $W_0$ , using Corollary 2.5, there is a blue path  $Q = \mathcal{P}'$ , with no  $v_{2n-1}$  as a vertex, of length  $l' = 2q'$  between

$\bar{P}' = e_2 e_3 \dots e_{n-1-r}$  and  $W'$  for some  $r \geq 0$  and  $W' \subseteq W_0$  with the mentioned properties in Corollary 2.5. Let  $y, z$  be the end vertices of  $Q$  in  $W'$ ,  $T = W_0 \setminus W'$  and  $x = |T|$ . We have one of the following cases.

**Case 1.**  $x = 0$ .

It is easy to see that  $l' = 2\lfloor \frac{m+1}{2} \rfloor - 2$ . If  $m$  is odd,  $l' = m - 1$  and clearly  $Q\{y, v_1, u\}$  is a blue copy of  $\mathcal{P}_m^3$ . If  $m$  is even,  $l' = m - 2$ . Since  $v_{2n-1}$  is not a vertex of  $Q$  and there is no copy of red  $\mathcal{C}_n^3$ , clearly  $\{v_1, u, y\}Q\{z, v_2, v_{2n-1}\}$  is a blue copy of  $\mathcal{P}_m^3$ .

**Case 2.**  $x = 1$ .

Let  $T = \{v\}$ . In this case,  $l' = 2\lfloor \frac{m+1}{2} \rfloor - 4$ . Clearly for odd  $m$ ,  $l' = m - 3$  and  $r \geq 1$ . One can easily check that  $Q\{z, v_2, v_{2n-2}\}\{v_{2n-2}, u, v\}\{v, v_{2n-1}, v_1\}$  is a blue  $\mathcal{P}_m^3$ . For even  $m$ , clearly  $l' = m - 4$  and  $r \geq 2$ . Since  $\hat{\mathcal{P}} = e_{n-2}e_{n-1}$  is maximal w.r.t.  $\bar{W} = \{y, z, u, v\}$ , by using Lemma 2.3, there is a blue  $\varpi_S$ -configuration, say  $\mathcal{P}''$ , with  $S \subseteq V(e_{n-2}) \cup V(e_{n-1})$  so that  $Q' = Q \cup \mathcal{P}''$  is a blue path of length  $m - 2$  and at least one of  $v_{2n-2}$  and  $v_{2n-1}$ , say  $w$ , is not in  $V(Q')$ . Without lose of generality assume that  $y$  and  $v$  are the end vertices of  $Q'$ . So  $\{u, v_1, y\}Q'\{v, w, v_2\}$  is a blue  $\mathcal{P}_m^3$ .

**Case 3.**  $x \geq 2$ .

One can easily check that  $r \geq 3$ . This case does not occur by Corollary 2.5.  $\blacksquare$

Using Theorem 2.2 we have  $R(\mathcal{P}_4^3, \mathcal{P}_4^3) = 10$ . Since  $R(\mathcal{P}_4^3, \mathcal{P}_3^3) \leq R(\mathcal{P}_4^3, \mathcal{P}_4^3)$ , we have the following Lemma.

**Lemma 4.2**  $R(\mathcal{P}_4^3, \mathcal{P}_3^3) = 10$ .

**Theorem 4.3** For every  $n \geq m \geq 3$ ,

$$R(\mathcal{P}_n^3, \mathcal{P}_m^3) = 2n + \left\lfloor \frac{m+1}{2} \right\rfloor.$$

**Proof.** We give a proof by induction on  $m + n$ . Using Theorem 2.2 the base case is trivial. Let  $\mathcal{H} = \mathcal{K}_{2n+\lfloor \frac{m+1}{2} \rfloor}^3$  be 2-edge colored red and blue with no a red copy  $\mathcal{P}_n^3$  of and a blue copy of  $\mathcal{P}_m^3$ . Consider the following cases.

**Case 1.**  $n = m$

By Theorem 2.2 and Lemma 4.2 we may assume  $n \geq 5$ . By the induction hypothesis,  $R(\mathcal{P}_{n-1}^3, \mathcal{P}_{n-1}^3) = 2(n-1) + \left\lfloor \frac{n}{2} \right\rfloor < 2n + \left\lfloor \frac{n+1}{2} \right\rfloor$ . So we may assume that there is a red copy of  $\mathcal{P}_{n-1}^3$ . Using Lemma 4.1 we have a blue copy of  $\mathcal{P}_n^3$  in  $\mathcal{H}$ , a contradiction.

**Case 2.**  $n > m$ .

In this case,  $n - 1 \geq m$ . Since  $R(\mathcal{P}_{n-1}^3, \mathcal{P}_m^3) = 2(n-1) + \left\lfloor \frac{m+1}{2} \right\rfloor < 2n + \left\lfloor \frac{m+1}{2} \right\rfloor$ , we may assume there is a copy of  $\mathcal{P}_{n-1}^3$  in  $\mathcal{H}_{red}$ . Using Lemma 4.1 we have a blue copy of  $\mathcal{P}_m^3$  in  $\mathcal{H}$ , a contradiction. These observations complete the proof.  $\blacksquare$



## 5 Path-cycle Ramsey numbers in 3-uniform hypergraphs

In this section, the Ramsey number of a loose path and a loose cycle in 3-uniform hypergraphs is determined.

It is worth noting that we can also conclude  $R(\mathcal{P}_n^3, \mathcal{C}_m^3) \leq n + \lfloor \frac{m+1}{2} \rfloor$ , for  $n \geq m \geq 3$ . To see that assume  $\mathcal{H} = \mathcal{K}_{2n+\lfloor \frac{m+1}{2} \rfloor}^3$  be 2-edge colored red and blue with no a red copy of  $\mathcal{P}_n^3$  and no a blue copy of  $\mathcal{C}_m^3$ . Since, using Theorem 3.3,  $R(\mathcal{C}_n^3, \mathcal{C}_m^3) = 2n + \lfloor \frac{m-1}{2} \rfloor < 2n + \lfloor \frac{m+1}{2} \rfloor$ , we have a red copy of  $\mathcal{C}_n^3$  in  $\mathcal{H}$ . Therefore, Lemma 2.1 implies a contradiction to our assumptions. So we have the following theorem.

**Theorem 5.1** *For every  $n \geq m \geq 3$ ,*

$$R(\mathcal{P}_n^3, \mathcal{C}_m^3) = 2n + \left\lfloor \frac{m+1}{2} \right\rfloor.$$

Combining Theorems 3.3, 4.3 and 5.1 yields a positive answer to the question 1.2. In the sequel, we determine  $R(\mathcal{P}_m^3, \mathcal{C}_n^3)$  when  $n > m \geq 3$ .

**Lemma 5.2**  $R(\mathcal{P}_3^3, \mathcal{C}_4^3) = 9$ .

**Proof.** Using Theorem 2.2 we have  $R(\mathcal{C}_4^3, \mathcal{C}_4^3) = 9$ . On the other hand  $R(\mathcal{P}_3^3, \mathcal{C}_4^3) \leq R(\mathcal{C}_4^3, \mathcal{C}_4^3)$ .  $\blacksquare$

**Theorem 5.3** *For every  $n > m \geq 3$ ,*

$$R(\mathcal{P}_m^3, \mathcal{C}_n^3) = 2n + \left\lfloor \frac{m-1}{2} \right\rfloor.$$

**Proof.** We prove the theorem by induction on  $m+n$ . By Lemma 5.2 the base case is trivial. Suppose to the contrary that  $\mathcal{H} = \mathcal{K}_{2n+\lfloor \frac{m-1}{2} \rfloor}^3$  is 2-edge colored red and blue with no a red copy of  $\mathcal{P}_m^3$  and no a blue copy of  $\mathcal{C}_n^3$  in  $\mathcal{H}$ . Consider the following cases.

**Case 1.**  $n = m+1$

By Theorem 5.1 we have  $R(\mathcal{P}_m^3, \mathcal{C}_m^3) = 2m + \left\lfloor \frac{m+1}{2} \right\rfloor < 2(m+1) + \left\lfloor \frac{m-1}{2} \right\rfloor$ . Since there is no red copy of  $\mathcal{P}_m^3$ , we have a blue  $\mathcal{C}_m^3$ . Now, by using Lemma 3.1 there is a red copy of  $\mathcal{P}_m^3$ , a contradiction.

**Case 2.**  $n > m+1$ .

By the induction hypothesis  $R(\mathcal{P}_m^3, \mathcal{C}_{n-1}^3) = 2(n-1) + \left\lfloor \frac{m-1}{2} \right\rfloor < 2n + \left\lfloor \frac{m-1}{2} \right\rfloor$ . Since there is no red copy of  $\mathcal{P}_m^3$ , we have a copy of  $\mathcal{C}_{n-1}^3$  in  $\mathcal{H}_{blue}$ . So using Lemma 3.1 we have a red copy of  $\mathcal{P}_m^3$ , a contradiction.  $\blacksquare$

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